STRESSES IN SOILS DUE TO VERTICAL LOAD ON SINGLE PILE AND PILE GROUP

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CERTIFICATE

Certified that work presented in this thesis has been carried out by Shri Virendra Singh under our supervision and has not been submitted elsewhere for a degree.

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DEDICATION

To my wife, Kokila Singh

NOTATIONS

D - Length of the pile.

a - Radius of the pile.

r - Radial distance of the point under consideration for stress from axis of the pile.

x - Number of interval of the pile.

v - Length of the one interval of the pile.

Ai - Vertical displacement of any interval mid point i.

dij - Vertical displacement of the soil at any location i due to unit force at location j.

dij' - deflection of the pile at any point i due to unit load on the pile at j.

F - Interaction force.

P - Force.

Z - Distance of the point under consideration for stress from the surface of the soil media.

Z' - Distance of the point under consideration for stress from image surface.

C' - Distance of the force P from surface of the soil media.

μ - Poisson's ratio.

G - Shear modulus.

R_1 - Distance of point under consideration for stress from the point of action of load on the shaft of the pile.

R_2 - Distance of point under consideration for stress from the image point of action of load corresponding to the point on shaft of the pile.

R_1' - Distance of point under consideration for stress from the point of action of load on the base of the pile.

R_2' - Distance of point under consideration for stress from the image point of action of the load corresponding to the point on base of the pile.

R_3 - Distance of point under consideration for stress from the inverted point of action of load.

R_4 - Distance of point under consideration for stress from the inverted image point of action of load.

w - Vertical displacement of the point.

w' - Additional vertical displacement.

w" - Vertical displacement of mid point i.

P' - Ring force.

Qn - Load on nth segment of the pile.

T - Tip load.

Y_T - Tipmovement.

S_3T - Load transfer in bottom segment.

P_f - Load transfer through shaft of the pile.

P_b - Load trransfer through base of the pile

p_f - Intensity of pressure of shaft load.

p_b - Intensity of pressure of base load.

- Angle measure from centre axis of the pile.

r' - Distance of any point on the base of the pile measure from centre of central axis of the pile.

dr' - Incremental thickness of base ring.

dh - Incremental thickness of shaft ring.

$\overline{zz}$ - Vertical stress.

$\overline{zz}_1$ - Vertical stress due to shaft load.

$\overline{zz}_2$ - Vertical stress due to base load.

$\overline{rr}$ - Radial stress.

$\overline{rr}_1$ - Radial stress due to shaft load.

$\overline{rr}_2$ - Radial stress due to base load.

$\overline{qq}$ - Circumferential stress.

$\overline{qq}_1$ - Circumjferential stress due to shaft load.

$\overline{qq}_2$ - Circumferential stress due to base load.

$\overline{rz}$ - Shear stress.

$\overline{rz}_1$ - shear stress due to shaft load.

$\overline{rz}_2$ - Shear stress due to base load.

$m = r/a$, $n = z/a$, $\alpha = D/a$, $\beta = h/D$, $\psi = r'/a$, $S = r/D$, $Q = z/D$.

u - Pore pressure.

du - Change in pore pressure.

A - Pore pressure coefficient.

B - Pore pressure coefficient.

$d\sigma_1$ - Change in major principal stress.

$d\sigma_3$ - Change in minor principal stress.

xk zz_1- Stress coefficient for shaft load.

xk zz_2- Stress coefficient for base load.

xk zzT- Stress coefficient for friction pile or for bearing pile.

Pk zzT- Geddes stress coefficient for friction pile.

Xi - X co-ordinate of the i the pile.

Yi - Y co-ordinate of the i the pile.

Xk - X co-ordiname of point under consideration for stress.

Yk - Y co-ordinate of point under consideration for stress.

Z - Z co-ordinate of point under consideration for stress.

N_1 - Number of parts of the limit interval.

C_1 - Facter deciding the nature of the pile.

C_1 - 1, means pile is totally friction pile.

C_1 - 0 means pile in totally end bearing pile.

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CHAPTER I

INTRODUCTION

1.1 GENERAL:

Piles transfer the load from a footing to the soils. Stress developed in the soil should not exceed the permissible value for the safety of the structure. Estimates of the consolidation settlement are commonly based on these calculated values of the stress. Therefore the dependability of such estimates is directly dependent on the accuracy with which the stresses are calculated. Pile foundation is generally used to transfer the load from heavy structure to subsurface soil. Load has been assumed to act axially on the pile. Pile is assumed to transfer the load by skin-friction and bearing area at the base. Results are presented in terms of dimensionless stress coefficients. These coefficients mainly vertical stress coefficients have been presented in a tabular form. Some results have been given for radial stress coefficients and other stresses can be obtained in a similar way. A brief literature review regarding the various aspects of load transfer is given in Chapter II. Stress at any point is a combined effect of load, partly transferred by the shaft and partly by the base of the pile. When it is assumed that the load transfer for the base of the pile is zero, Pile becomes friction pile. For the friction pile the vertical stress coefficients have been compared with the results of Geddes (1966). A pile which transmits the load only through the base becomes a bearing pile. Tables

are given for this case also. By knowing the load transmitted by the skinfriction and by the bearing area, the actual stress can be obtained by simple multiplication of these loads with the corresponding stress coefficient. This is discussed in Chapter III for an axially loaded single pile. Chapter IV gives the stresses in soil due to vertical load on a group of pile. and given

1.2 SCOPE OF THE PRESENT WORK:

Solution given in Chapter III is more precise than earlier methods for computing stresses in soils. In this thesis solutions have been obtained for the stresses created by vertical loading on a single pile or a pile group by Euler's formula of summation using solutions given by Mindlin (1936) for a point load within a semi-infinite medium.

CHAPTER II

LITERATURE REVIEW

2.1 INTRODUCTION:

Estimation of the stress distribution in soils due to vertical loads is largely based on the work of Boussinesq (1885) and Mindlin (1936). Boussinesq considered the case of vertical point load applied at the surface of a semi-infinite, isotropic and homogeneous medium obeying Hook's law where as Mindlin considered the case of a vertical point load acting below the surface of a semi-infinite medium. Although few of the assumptions regarding the properties of the medium are totally valid in the case of soils, experience has shown that the calculated values give a useful indication of the order of the stresses and their variation from point to point.

In practice few foundations apply their load at the ground surface yet Boussinesq equation has been used to find out the stresses caused by the subsurface loading. In case of piles, Terazaghi (1943) proposed a simpler method to find out stresses at a point due to shaft loading by numerical integration of the Boussinesq equation for a point load . To find out the stress due to shaft loading of a pile, a better method will be to integrate the Boussinesq equation for point load mathematically. Geddes (1966, 1969) has done the mathematical integration based on Mindlin and Boussinesq equation for point load respectively.

Literature review has been done for load trasfer, calculation of the stress and the effect of applied stress on pore pressure and consolidation.

2.2 LOAD TRANSFER THROUGH A PILE:

Piles receive their support in the form of shaft load from the side of the pile and tip load at the bottom of the pile. Friction pile is one where the tip load is small in comparison to the shaft load. End bearing pile is one where the shaft load is small in comparison to the tip load.

D Appolonia and Romaldi (1963) have presented a mathematical analysis of load transfer through pile. Following assumptions have been made. The tip of the pile is assumed not to move. The soil trapped between the flanges of the pile is assumed to act integrally with the pile and the surrounding soil is assumed to be a semi infinite elastic solid.

The theoretical load transfer between a point bearing steel pile and an elastic medium can be calculated from fundamental compatibility concepts in the theory of elasticity. An end bearing pile of length D is embeded in soil (Figure 2.1). Pile is divided in x equal parts having length v. The interaction shear stress between the pile and soil is assumed to be constant over the length v and the resultant force F is assumed to act at the mid point of the interval. Pile is free to move within the soil. Let Δi be the vertical displacement of any interval mid point i. This is the displacement of any interval mid point i of the pile relative to the soil. But for calculating interaction forces assumption is made that there is no relative motion between pile and soil. The interaction force may be assumed to be the force of the soil on the pile (negative upward) or its equal and opposite reaction, the force of the pile on the soil.

Let d_{ij} be the vertical displacement of the soil at any location i due to unit force at location j. d'_{ij} is the deflection of the pile at any point i due to unit load on the pile at j. Then the condition that the interaction forces F be of such magnitude that there be no relative displacement between the pile and soil at any position i is then

$$\sum_{j=1}^{M} d_{ij} \; F_j + \sum_{j=1}^{M} d'_{ij} \; F_j \qquad = \Delta i \qquad (2.1)$$

or

$$\sum_{j=1}^{M} (d_{ij} + d'_{ij}) \; F_j \qquad = \Delta i$$

This leads to a system of M simultaneous equation for the forces F_1, F_2 ---- F_M.

d_{ij} is calculated by use of the Mindlin equation. The vertical displacement at depth Z due to force P at a distance C' from the free surface is

$$w = \frac{P}{16 \pi G(1-\mu)} \left(\frac{(3-4\mu)}{R_1} + \frac{8(1-\mu)^2 - (3-4\mu)}{R_2} + \frac{(Z-C')^2}{R_1^3} \right.$$

$$\left. + \frac{(3-4\mu)\;(Z+C')^2 - 2\,C'Z}{R_2^3} + \frac{6\,C'Z\;(Z+C')^2}{R_2^5} \right) \qquad (2.2)$$

Equation (2.2) assumes a semi-infinite media and in case of an end bearing pile there is a restraint at depth D. A surface at depth D can be assumed to be a surface of zero vertical displacement. This condition can be analytically approximated by adding a mirror image as shown in Figure 2.2.

Then the displacement given by equation (2.2.) must be corrected by the addition of w'.

$$w' = \frac{P}{16\pi G(1-u)}\left(\frac{(3-4u)}{R_3} + \frac{8(1-u)-(3-4u)}{R_4} + \frac{(Z'-C')^3}{R_2^3}\right.$$

$$\left. + \frac{(3-4u)\ (Z'+C')-2\ C'Z'}{R_4^3} + \frac{6\ C'\ Z'\ (Z'+C')^2}{R_4^5}\right) \qquad (2.3)$$

dij is obtained by the addition of equations (2.2) and (2.3) with the appropriate value of Z and C' corresponding to i and J respectively. The above method is not valid when Z = C' due to stress siggularity at such points. An approximate solution can, however, be obtained by assuming the pile to be cylindrical and then the interaction stress is assumed to be uniform over the interval h. The desired displacement w" at the mid point of i along the centre of the pile due to the distributed unit stress is given by

$$w'' = \frac{1}{2\pi a v} \qquad (2.4)$$

Due to axial symmetry a solution is obtained by assuming ring force $P' = w''\ (2\pi a)\ d\xi$ (2.5)

Where $d\xi$ is the thickness of small ring load. To avoid the complication arising from equation (2.2) it is assumed that the total force acts around the circumference.

Reese (1966) has presented a load - settlement curve to determine the load transfer by different segments of the pile and his method is explained by the aid of (Figure 2.3).

In this method it is desired to compute the load Q_o and δ at the top of the pile. Assuming small tip movement at the bottom segment, force and movement of each segment is calculated. Thus for a particular tip movement of the bottom segment Q_o and δ is foundout. For different assumed tip movements different values of Q_o and δ will be obtained and a load - settlement curve can be plotted. In figure 2.3, Q_o, Q_1, Q_2, Q_3 are loads on corresponding segments.

$$Q_3 = S_3T + T \qquad (2.6)$$

T = tip load

Y_T = tipmovement

S_3T = load transfer in bottom segment

knowing the way load is transferred, making certain assumptions, Boussinesq and Mindlin solutions have been used by Geddes to calculate the stresses in soil.

2.3 BOUSSINESQ SOLUTION:

The equations expressing the stress components caused by vertical point load applied at the surface of a semi-infinite, isotropic and homogeneous medium are given as

$$\overline{zz} = \frac{P}{2\pi} \frac{3\,z^3}{(r^2+z^2)5/2}$$

$$\overline{rr} = \frac{P}{2\pi} \left(\frac{3\,r^2 z}{(r^2+z^2)5/2} - \frac{(1-2u)}{(r^2+z^2+z(r^2+z^2)\tfrac{1}{2})} \right.$$

$$\overline{qq} = -\frac{P\,(1-2u)}{2\pi} \left(\frac{z}{(r^2+z^2)3/2} - \frac{1}{(r^2+z^2+z(r^2+z^2)\tfrac{1}{2})} \right.$$

$$\overline{rz} = \frac{P}{2\pi} \frac{3\,r\,z^2}{(r^2+z^2)5/2} \qquad (2.7)$$

Solution for vertical stress for a point load acting at a distance D from the surface is arrived at from equation (2.7) neglection over burden as (Geddes)

$$\overline{zz} = \frac{3P}{2\pi} \quad \frac{(z - D)^3}{((r^2 + (z - D)^2)^{5/2}} \qquad (2.8)$$

Geddes has non dimensionalised the equation (2.8) by putting S=r/D and Q=z/D and calculated stress coefficient KB'= stress D^2/P. Results have been presented in Tabular form.

$$KB' = -\frac{3}{2\pi} \quad \frac{(Q-1)^3}{(S^2+(Q-1)^2)^{5/2}} \qquad (2.9)$$

2.4 Mindlin Solution:

For a point load applied at depth D below the surface in an isotropic media, the various stresses given by Mindlin are

$$\overline{zz} = \frac{P}{8\pi(1-\mu)} \left(-\frac{(1-2\mu)\ (z - D)}{R_1^3} + \frac{(1-2\mu)\ (z - D)}{R_2^3} - \frac{3(z - D)^3}{R_1^5} \right.$$

$$- \frac{(3(3 - 4\mu)\ z\ (z + D)^2 - 3\ D(z + D)(5\ z - D)}{R_2^5}$$

$$\left. - \frac{30\ z\ D\ (z + D)^3}{R_2^7} \right) \qquad (2.10)$$

$$\overline{rr} = \frac{P}{8\pi(1-\mu)} \left(\frac{(1-2\mu)(z-D)}{R_1^3} - \frac{(1-2\mu)(z + 7\ D)}{R_2^3} - \frac{3r^2(z-D)}{R_1^5} \right.$$

$$+ \frac{4(1-\mu)(1-2\mu)}{R_2(R_2+Z+D)} - \frac{3r^2(z+D)\ 10\ z\ D}{R_2^7}$$

$$+ \frac{6\ D(1-2\mu)(z + D)^2 - 6\ D^2(z+D) - 3(3-4\mu)\ r^2(z-D)}{R_2^5} \qquad (2.11)$$

$$\overline{qq} = \frac{P}{8\pi(1-u)} \left(\frac{(1-2u)(z-D)}{R_1^3} + \frac{(1-2u)(3-4u) - 6D(1-2u)}{R_2^3} \right.$$

$$\left. - \frac{4(1-u)(1-2u)}{R_2(R_2+z+D)} + \frac{(1-2u)\ 6D(z+D)^2 - 6D^2(z+D)}{R_2^5} \right) \qquad (2.12)$$

$$\overline{rz} = \frac{Pr}{8\pi(1-u)} \left(-\frac{(1-2u)}{R_1^3} + \frac{(1-2u)}{R_2^3} - \frac{3(z-D)^2}{R_1^5} - \frac{30\ zD(z+D)^2}{R_2^7} \right.$$

$$\left. - \frac{(3(3-4u)\ z(z+D) - 3D(3\ z + D))}{R_2^5} \right) \qquad (2.13)$$

in which

$$R_1^2 = r^2 + (z - D)^2 \qquad (2.14)$$

$$R_2^2 = r^2 + (z + D)^2 \qquad (2.15)$$

see figure 2.5.

2.5 G.D. GEDDES SOLUTION FOR VARIOUS STRESSES DUE TO DIFFERENT TYPES OF LOADING:

Using Mindlin's equations the stresses have been found out by Geddes (1966) for point load, uniform skin friction and linear variation of skin friction. Vertical stress due to uniform skin friction is found out as:

The incremental load over depth dh will be dp given by

$$dp = (P/D)\,dh \qquad (2.16)$$

stress due to total load P is given by

$$zz = \left(\frac{P}{D}\right) \frac{1}{8\pi(1-u)} \int_0^D \left(-\frac{(1-2u)(z-h)}{C^3} + \frac{(1-2u)(z-h)}{E^3} - \frac{3(z-h)^3}{C^5} \right.$$

$$\left. - \frac{30\ hz(z+h)^3}{E^7} - \frac{(3(3-4u)\,z(z+h)^2 - 3h(z+h)(5z-h)}{E^5} \right) dh \qquad (2.17)$$

in which

$$C^2 = (r^2 + (z-h)^2) \tag{2.18}$$

$$E^2 = (r^2 + (z+h)^2) \tag{2.19}$$

Vertical stress due to linear variation of skin friction:

Load per unit depth $= 2P \frac{h}{D^2}$

Force applied over depth dh is

$dp = 2P \frac{h}{D^2} dh$ then the vertical stress

$$\sigma_{zz} = \frac{P}{4\pi(1-\mu)} \int_0^D \Big(-\frac{(1-2\mu)(z-h)h}{C^3} + \frac{(1-2\mu)(z-h)h}{E^3} - \frac{3h(z-h)^3}{C^5}$$

$$- \frac{(3(3-4\mu)zh(z+h)^2 - 3h^2(z+h)(5z-h))}{E^5} - \frac{30zh^2(z+h)^3}{E^7} \Big) dh \tag{2.20}$$

The stress due to pile loading may be causing an increase in pore pressure which is important for the study of the consolidation of soil strata. These are briefly reviewed below.

2.6 DEVELOPMENT OF PORE PRESSURE IN SOILS BY APPLIED STRESSES:

In the case of clays it is of interest to compute the instantaneous excess pore-water pressure distribution in the soil due to the applied stress. With time, this fluid stress will dissipate, throwing increasing amounts of the applied stress on the soil skelton in the form of effective pressures, with resulting increasing settlement with time. Eventually all of the applied stresses are carried by the soil structure. Expression for excess pore pressure is given by Skempton (36) as

$$du = B\,(d\sigma_3 + A(d\sigma_1 - d\sigma_3)) \tag{2.21}$$

Where A and B are the pore pressure coefficients. A and B are not constant but vary with the amount of strain which takes place in the sample. Skempton has given the chart for the values of A and B for different soils. It should be kept in mind that

the pore pressure generated by an applied stress system depends on the way in which the final stress state is reached.

2.7 CONSOLIDATION DUE TO STRESS:

In the vicinity of the pile, stresses are predominant by load on pile. Due to these applied stresses excess pore water pressure is developed which dissipates with time, resulting in settlement of the soil near the pile. Due to this consolidation of the soil, interaction forces on the pile may be altered.

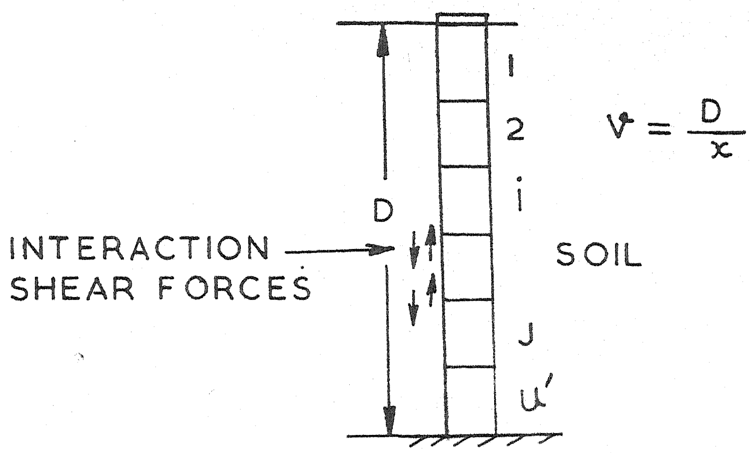


FIG. 2.1

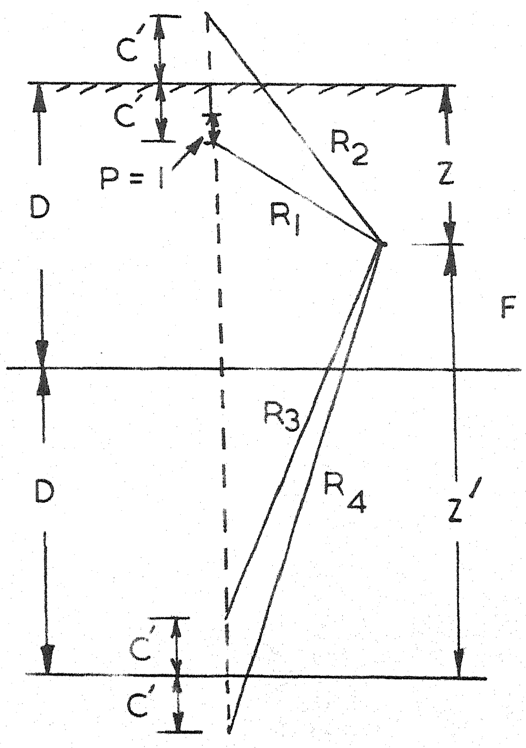


FIG. 2.2 EFFECT OF FORCE P IN PRESENCE OF RIGID - BOUNDARY AT DEPTH D

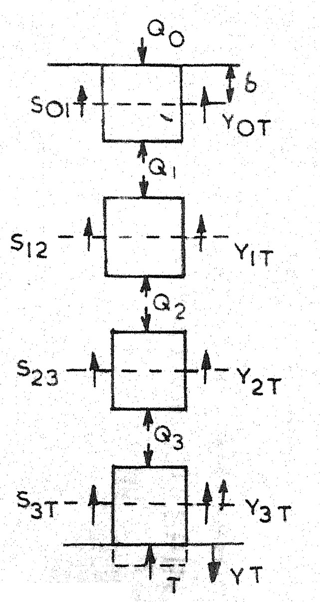


FIG. 2.3 AXIALLY LOADED PILE - SHOWING FORCES ACTING ON SEGMENT OF THE PILE

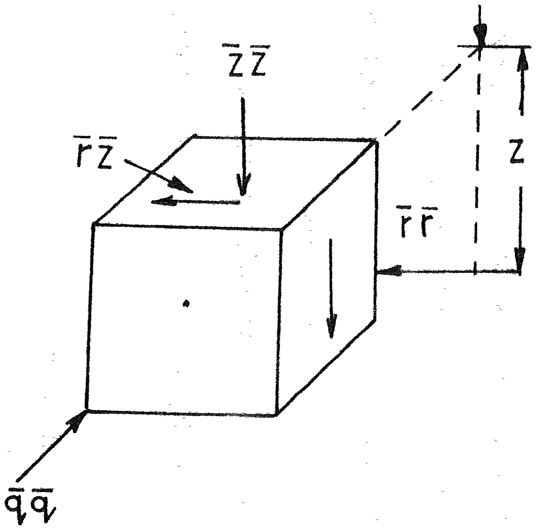


FIG. 2·4 STRESSES IN CYLINDRICAL CO ORDINATES CAUSED BY A SURFACE VERTICAL POINT LOAD

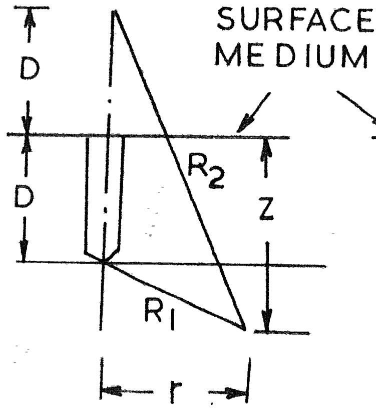


FIG. 2·5 POINT LOAD

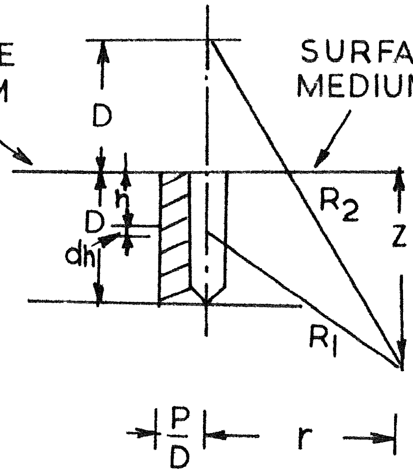


FIG. 2·6 UNIFORM SKIN FRICTION

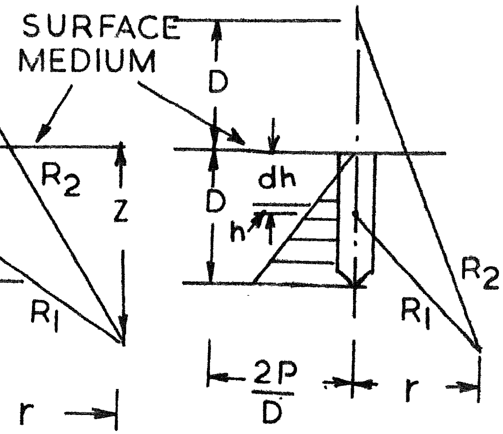


FIG. 2·7 LINEAR VARIATION OF SPIN FRICTION

CHAPTER III

ANALYSIS OF STRESS IN SOILS DUE TO VERTICAL LOAD ON SINGLE PILE

3.1 INTRODUCTION:

In an actual situation, the pile transfers its load through the shaft and through the base. At any point the stress produced is a combined effect of these two loads transferred. Till now no mathematical solution has been presented taking these considerations into account. Geddes takes only shaft load into account and that too assuming the pile as a line neglecting the effect of diameter of the pile.

FORMULATION OF THE PROBLEM

3.2 BASIC EQUATIONS:

In this investigation it is assumed that the shaft load gets transferred to surrounding soils as ring load. The intensity of pressure pf for shaft load and pb for base load are assumed uniform. If P is the load acting on the pile then

$$P = Pf + Pb \qquad (3.1)$$

Pf = load transferred by the pheriphery of the pile.

Pb = load transferred by the base of the pile.

$$Pf = pf \int_0^D \int_0^{2\pi} a \, d\theta \, dh \qquad (3.2)$$

$$Pb = pb \int_0^a \int_0^{2\pi} r' \, d\theta \, dr' \qquad (3.3)$$

pf = intensity of the pressure on the pheriphery of the pile

pb = intensity of the pressure on the base of the pile.

For above mentioned parameters see Fig. 3.1 and Fig. 3.2.

3.3 STRESSES DUE TO SHAFT LOAD:

Shaft load gets transferred to the soils as a ring load. So the distance between the point under consideration in soil media and the points on the periphery of the ring load is not constant. It is varying from (r-a) to a maximum of (r+a) where a is the radius of the pile and r is the horizontal distance of the point under consideration from vertical axis of the pile. Suppose point P_1 is taken into consideration on pile periphery and S' in soil media as shown in figure 3.3.

$AS' = r$

$AB = a \cos\theta$

$BS' = r - a \cos\theta$

$P_1B = a \sin\theta$

$$P_1S' = ((r-a\cos\theta)^2+(a \sin\theta)^2)^{1/2} \qquad (3.4)$$

Equation (3.4) is a general equation which takes into account the position of various points on pile. Due to shaft load through pile, vertical, radial, circumferential and shearing stresses at a point defined by cylindrical co-ordinates (r, θ, z) are given by using equation 3.2.

$$\overline{zz}_1 = \frac{apf}{8\pi(1-\mu)} \int_0^D \int_0^{2\pi} \Big(-\frac{(1-2u)(z-h)}{R_1^3} + \frac{(1-2u)(z-h)}{R_2^3} - \frac{3(z-h)^3}{R_1^5}$$

$$- \frac{30\,hz(z+h)^3}{R_2^7} - \frac{(3(3-4u)z(z+h)^2-3h(z+h)(5z-h)}{R_2^5} \Big) d\theta\, dh \qquad (3.5)$$

$$\overline{rr}_1 = \frac{apf}{8\pi(1-\mu)} \int_0^D \int_0^{2\pi} \Big(\frac{(1-2u)(z-h)}{R_1^3} - \frac{(1-2u)(z+7h)}{R_2^3} + \frac{4(1-u)(1-2u)}{R_2(R_2+z+h)}$$

$$- \frac{3r^2(z-h)}{R_1^5} - \frac{30Dr^2z(z+h)}{R_2^7} + \frac{6D(1-2u)(z+h)^2-6D^2(z+h)-3r^2(3-4u)(z-h)}{R_2^5} \Big) d\theta dh$$

(3.6)

$$\overline{qq} = \frac{a\,bf}{8\pi(1-\mu)} \int_0^D \int_0^{2\pi} \Big(\frac{(1-2u)(z-h)}{R_1^3} + \frac{(1-2u)(3-4u)(z+h)-(1-2u)\,6h}{R_2^3}$$

$$- \frac{4(1-\mu)(1-2u)}{R_2(R_2+z+h)} + \frac{(1-2u)\,6h(z+h)^2 - 6h^2(z+h)}{R_2^5} \Big)\, d\theta dh \qquad (3.7)$$

$$\overline{rz} = \frac{a\,r\,bf}{8\pi(1-u)} \int_0^D \int_0^{2\pi} \Big(-\frac{(1-2u)}{R_1^3} + \frac{(1-2u)}{R_2^3} - \frac{3z(z-h)^2}{R_1^5} - \frac{30zh(z+h)^2}{R_2^7}$$

$$- \frac{(3(3-4u)\ z\ (z+h) - 3h\ (3z+h))}{R_2^5} \Big)\, d\theta\, dh \qquad (3.8)$$

Where

$$R_1^2 = ((r - a\operatorname{Cos}\theta)^2 + (a\operatorname{Sin}\theta)^2 + (z-h)^2)$$

$$R_2^2 = ((r - a\operatorname{Cos}\theta)^2 + (a\operatorname{Sin}\theta)^2 + (z+h)^2)$$

To compute the vertical stress equation 3.5 has been non-dimensionalised. Let

$$\frac{r}{a} = m, \quad \frac{z}{a} = n, \quad \frac{D}{a} = \alpha, \quad \frac{h}{D} = \beta$$

$$h/a = B$$

$$dh = Dd$$

Limit of h is 0 to D

So limit of B is 0 to 1.

Now,

$$R_1 = a(m^2 + n^2 + \alpha^2 B^2 - 2\,m\operatorname{Cos}\theta - 2\,n\,\alpha\beta + 1)^{1/2}$$

$$R_2 = a(m^2 + n^2 + \alpha^2 B^2 - 2\,m\operatorname{Cos}\theta + 2\,n\,\alpha\beta + 1)^{1/2}$$

$$\overline{zz}_1 = \frac{bf}{8\pi(1-\mu)} \int_0^1 \int_0^{2\pi} \Big(- \frac{a^2(1-2u)(\frac{z}{a}-\frac{h}{a})}{R_1^3} + \frac{a^2(1-2u)(\frac{z}{a}-\frac{h}{a})}{R_2^3}$$

$$- \frac{3a^4(\frac{z}{a}-\frac{h}{a})^3}{R_1^5} - \frac{30a^6\ \frac{h}{a}\ \frac{z}{a}\ (\frac{z}{a}+\frac{h}{a})^3}{R_2^7}$$

$$\frac{(3a^4(3-4u)\frac{z}{a}(\frac{z}{a}+\frac{h}{a})^2 - 3a^3\,\frac{h}{a}\,(\frac{z}{a}+\frac{h}{a})\,a(\frac{5z}{a}-\frac{h}{a})}{R_2^5} \Big)\, d\theta\, D\, dB$$

$$\overline{RZ}_1 = \frac{bf}{8(1-u)} \int_0^1 \int_0^{2\pi} \Big(- \frac{\alpha(1-2u)(n-\alpha p)}{(m^2+n^2+\alpha^2p^2-2m\cos\theta-2n\alpha p+1)^{3/2}}$$

$$+ \frac{\alpha(1-2u)(n-\alpha p)}{(m^2+n^2+\alpha^2p^2-2m\cos\theta+2n\alpha p+1)^{3/2}}$$

$$- \frac{3\alpha(n-\alpha p)^3}{(m^2+n^2+\alpha^2p^2-2m\cos\theta-2n\alpha p+1)^{5/2}}$$

$$- \frac{30\alpha n\alpha p(n+\alpha p)^3}{(m^2+n^2+\alpha^2p^2-2m\cos\theta+2n\alpha p+1)^{7/2}}$$

$$- \frac{3\alpha(3-4u)n(n+\alpha p)^2 - 3\alpha\alpha p(n+\alpha p)(5n-\alpha p)}{(m^2+n^2+\alpha^2p^2\,2m\cos\theta+2n\alpha p+1)^{5/2}} \Big) d\theta\, dp$$

(3.9)

3.4 STRESSES DUE TO BEARING LOAD:

In this case load get transferred from the entire area of the base. Various stresses, vertical, radial, circumferential and shearing stresses produced at the point are given by the following equations. by using equation 3.3.

$$\overline{RZ}_2 = \frac{qb}{8\pi(1-u)} \int_0^a \int_0^{2\pi} \Big(- \frac{(1-2u)(z-D)}{R_1'^3} + \frac{(1-2u)(z-D)}{R_2'^3} - \frac{3(z-D)^3}{R_1'^5}$$

$$- \frac{30zD(z+D)^3}{R_2'^7} - \frac{3(3-4u)z(z+D)^2 - 3D(z+D)(5z-D)}{R_2'^5} \Big) r'\, d\theta\, dr'$$

(3.10)

$$\overline{RR}_2 = \frac{qb}{8\pi(1-u)} \int_0^a \int_0^{2\pi} \Big(\frac{(1-2u)(z-D)}{R_1'^3} - \frac{(1-2u)(z+7D)}{R_2'^3} + \frac{4(1-u)(1-2u)}{R_2'(R_2'+z+D)}$$

$$- \frac{3r^2(z-D)}{R_1'^5} - \frac{30Dr^2z(z+D)}{R_2'^7}$$

$$\frac{6D(1-2u)(z+D)^2 - 6D^2(z+D) - 3(3-4u)r^2(z-D)}{R_2'^5} \Big) r'\, d\theta\, dr' \qquad (3.11)$$

$$\overline{qq}_2 = \frac{pb}{8\pi(1-\mu)} \int_0^a \int_0^{2\pi} \Big(\frac{(1-2\mu)(z-D)}{R_1'^3} + \frac{(1-2\mu)(3-4\mu)(z+D) - (1-2\mu)6D}{R_2'^3}$$

$$- \frac{4(1-\mu)(1-2\mu)}{R_2'(R_2'+z+D)} + \frac{(1-2\mu)6D(z+D)^2 - 6D^2(z+D)}{R_2'^5} \Big) r'd \quad dr' \quad (3.12)$$

$$\overline{rz}_2 = \frac{pbr}{8\pi(1-\mu)} \int_0^a \int_0^{2} \Big(- \frac{(1-2\mu)}{R_1'^3} + \frac{(1-2\mu)}{R_2'^3} - \frac{3(z-D)^2}{R_1'^5} - \frac{30zD(z+D)^2}{R_2'^7}$$

$$- \frac{3(3-4\mu)z(z+D) - 3D(3z+D)}{R_2'^5} \Big) r'd\theta dr' \quad (3.13)$$

$$R_1'^2 = ((r - r' \cos\theta)^2 + (r' \sin\theta)^2 + (z - D)^2)$$

$$R_2'^2 = ((r - r' \cos\theta)^2 + (r' \sin\theta)^2 + (z + D)^2)$$

r' and θ is shown in figure 3.2, and figure 3.4.

$AM = a$, $AP_1 = r'$, $\angle P_1AB = \theta$

$$P_1C = ((r - r' \cos\theta)^2 + (r' \sin\theta)^2)^{\frac{1}{2}} \quad (3.14)$$

To compute the vertical stress due to bearing load equation 3.10 has been nondimensionalised.

Let $r'/a = \psi$

$dr' = a d\psi$

Limit of r' is 0 to a

So Limit of ψ is 0 to 1

$$R_1' = a(m^2 + n^2 + \alpha^2 + \psi^2 + 2m\psi\cos\theta - 2n\alpha)^{\frac{1}{2}}$$

$$R_2' = a(m^2 + n^2 + \alpha^2 + \psi^2 - 2m\psi\cos\theta + 2n\alpha)^{\frac{1}{2}}$$

$$\overline{zz}_2 = \frac{pb}{8\pi(1-\mu)} \int_0^a \int_0^{2\pi} \Big(- \frac{(1-2\mu)(z-D)}{R_1'^3} + \frac{(1-2\mu)(z-D)}{R_2'^3} - \frac{3(z-D)^3}{R_1'^5} - \frac{30zD(z+D)^3}{R_2'^7}$$

$$- \frac{3(3-4\mu)z(z+D)^2 - 3D(z+D)(5z-D)}{R_2'^5} \Big) r'd\theta dr'$$

$$\bar{zz}_2 = \frac{pb}{8\pi(1-\mu)}\int_0^1\int_0^{2\pi}\Big(-\frac{\psi(1-2\mu)(n-\alpha)}{(m^2+n^2+\alpha^2+\psi^2-2m\psi\cos\theta-2n\alpha)^{3/2}}$$

$$+\frac{\psi(1-2\mu)(n-\alpha)}{(m^2+n^2+\alpha^2+\psi^2-2m\psi\cos\theta+2n\alpha)^{3/2}}$$

$$-\frac{3\psi(n-\alpha)^3}{(m^2+n^2+\alpha^2+\psi^2-2m\psi\cos\theta-2n\alpha)^{5/2}}$$

$$-\frac{30\psi n\alpha(n+\alpha)^3}{(m^2+n^2+\alpha^2+\psi^2-2m\psi\cos\theta+2n\alpha)^{7/2}}$$

$$-\frac{3\psi(3-4\mu)n(n+\alpha)^2-3\psi(n+\alpha)(5n-\alpha)}{(m^2+n^2+\alpha^2+\psi^2-2m\psi\cos\theta+2n\alpha)^{5/2}}\Big)d\theta d\psi \quad (3.15)$$

3.5 VERTICAL STRESS BASED ON EULER'S FORMULA OF SUMMATION:

$$\bar{zz} = \bar{zz}_1 + \bar{zz}_2$$

$$= pf\ xk\ zz_1 + pb\ xk\ zz_2$$

$$= \frac{Pf}{2\pi aD}xk\ zz_1 + \frac{Pb}{\pi a^2}xk\ zz_2$$

$$= \frac{C_1P}{2\pi aD}xk\ zz_1 + \frac{(1-C_1)P}{\pi a^2}xk\ zz_2$$

$$= \frac{P}{a^2}\left(\frac{C_1}{2\pi\alpha}xk\ zz_1 + \frac{(1-C_1)}{\pi}xk\ zz_2\right) \quad (3.16)$$

$$=(P/a^2)\ xk\ zzT$$

$xk\ zz_1$, $xk\ zz_2$, $xk\ zzT$ are called stress coefficients. Once we know $xk\ zz_1$, $xk\ zz_2$ for a point and knowing C_1 we can calculate the stress. If C_1 is one it means pile is totally friction pile. If C_1 is zero it means the pile is totally end bearing pile. But in actual case pile is neither a friction pile nor an end bearing pile. Under field condition the value of C_1 has to be determined. Once the value of C_1 is

decided, stress can atonce be found by equation (3.16). For friction pile and end bearing pile for different value of μ stress coefficient xk zzT has been presented in Tabular form. S and Q has been introduced.

$$S = r/D, \quad Q = z/D$$

Where r is the distance of the point from the axis of the pile. z is the vertical distance of point from the surface. Table for radial stress is also given. Similarily others stresses can be computed.

3.6 RESULTS:

For different value of S, Q, D/a, and μ the value of xk zzT are given in Tabular form. See the Tables (3.1-3.23). To compare the values with Geddes values (1966) a new stress coefficient Pk zzT has been introduced. According to Geddes,

$$\text{stress} = \frac{P}{D^2} \text{ Pk zzT}.$$

According to investigation stress $= \frac{P}{a^2}$ xK zzT

$$\text{PK zzT} = \alpha^2 \text{ xk zzT} \qquad (3.17)$$

For different value of S, Q, μ, C_1, xk zzT has been Tabulated. A few graphs has been plotted between r/D and xk zzT for a fixed vertical plane.

3.7 CONCLUSION:

Comparing the stress coefficient value with Geddes it is found that Geddes theory under estimates the stresses. Difference near the vicinity of the pile is quite large, larger value is got for the solution obtained which is expected. With increasing D/a, the vertical stress is decreasing.

Accuracy of the method for evaluating the double integral is tested by taking the number of intervals 20, 40 and 80 respectively and examining the values of xk zzT with these intervals. It is observed that the Euler's summation method gives converging results (Table 3.23).

TABLE 3.1: VALUE OF xk zzT x 10° for μ = 0.0, C_1=1 and D/a = 20(+Tension, otherwise compression).

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	1845	908	436	232	135	84	54	36	24	17	12	8	6	4	3	2	2	1	1	1	0	0	0	0
.4	1290	1022	740	514	353	243	168	118	83	59	43	31	23	17	13	10	7	6	4	3	2	1	1	1
.6	1297	1104	876	666	495	365	268	197	146	108	81	61	46	45	27	21	17	13	10	8	5	4	2	2
.8	1955	1403	1009	747	563	428	327	251	193	144	116	90	71	55	44	35	28	22	18	15	10	7	5	3
1.0	3233	1565	1030	751	573	446	352	270	222	177	142	114	91	74	60	49	40	33	27	22	15	11	8	6
1.2	1535	1183	892	687	542	435	352	287	235	192	158	130	107	89	73	61	51	42	35	30	21	15	11	8
1.4	881	789	678	570	477	398	333	279	234	197	166	139	117	99	84	71	60	51	43	37	27	20	15	11
1.6	594	559	510	455	400	348	302	260	224	193	166	142	122	105	90	78	67	57	50	43	32	24	18	14
1.8	436	420	395	364	331	298	266	236	208	183	160	140	123	107	94	82	71	62	54	48	36	28	22	17
2.0	337	328	314	296	276	254	231	210	189	169	151	135	120	106	94	83	74	65	58	51	40	31	25	20
2.2	271	265	256	245	231	217	201	185	170	155	140	127	114	103	92	83	74	66	59	53	42	34	27	22
2.4	223	219	213	206	196	186	175	163	152	140	129	118	108	98	89	81	73	66	60	54	44	36	29	24
2.6	187	185	181	175	169	161	153	144	136	127	118	109	101	93	85	78	71	65	60	54	45	37	31	25
2.8	159	158	155	151	146	141	135	128	121	114	107	100	94	87	81	75	69	63	58	54	45	38	32	27
3.0	138	136	134	132	128	124	119	114	109	103	98	92	87	81	76	71	66	61	57	52	45	38	32	27

TABLE 3.2 VALUE OF xk zzT $\times 10^5$ for μ = 0.0, C_1=0.0 and D/a = 20 (+Tension, otherwise compression).

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	+ 162	+ 132	+ 94	+ 57	+ 28	+ 8	4	9	11	11	10	9	7	6	5	4	3	2	2	1	1	1	0	0
.4	+ 622	+ 483	+ 321	+182	+ 82	+ 20	14	30	35	35	31	27	23	19	15	12	10	8	7	5	3	2	2	1
.6	+1734	+1139	+ 611	+274	+ 89	4	46	62	65	61	55	47	40	34	28	23	19	16	13	11	7	5	3	2
.8	+5389	+1882	+ 559	+127	26	82	99	100	94	85	74	64	55	47	39	33	28	23	20	16	12	8	6	4
1.0	211	206	197	186	174	160	145	130	115	101	89	77	66	57	49	42	36	30	26	22	16	12	9	6
1.2	5819	2301	960	504	324	239	190	158	135	116	100	87	75	65	87	49	42	37	32	27	20	15	11	9
1.4	2187	1579	1029	663	446	319	242	192	158	132	113	97	84	73	64	55	48	42	37	32	24	19	14	11
1.6	1110	953	762	586	446	342	268	215	176	147	125	107	92	80	70	61	54	47	41	36	28	22	17	13
1.8	680	624	547	464	386	318	262	218	182	154	132	113	98	86	75	66	58	51	45	40	32	25	20	16
2.0	466	442	406	363	319	277	239	206	177	153	133	116	101	89	79	70	62	55	49	43	35	28	22	18
2.2	344	331	312	289	263	237	211	187	165	146	129	114	101	90	80	72	64	57	51	46	37	30	24	20
2.4	266	259	248	234	218	201	184	167	151	136	122	110	99	89	80	72	65	58	53	48	39	32	26	22
2.6	214	210	203	194	184	172	160	148	136	125	114	104	94	86	78	71	64	59	53	48	40	33	28	23
2.8	176	174	169	163	156	148	140	131	122	113	105	97	89	82	75	69	63	58	53	48	41	34	29	24
3.0	148	147	144	139	134	129	123	116	109	103	96	89	83	77	71	66	61	56	52	48	41	35	29	25

TABLE 3.3: VALUE OF xk $zz_1 \times 10^5$ for $\mu = 0.0$, D/a = 20 (+ Tension otherwise compression)

S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8
.2	23189	11416	5485	2917	1702	1056	682	452	305	209	145	102	73	52	38	38	21	16	12	9	5	3	2	1
.4	16213	12846	9296	6459	4436	3052	2115	1479	1044	744	536	390	286	212	159	120	92	70	55	43	27	17	12	8
.6	16303	13879	11004	8367	4224	4585	3369	2480	1833	1362	1079	767	582	445	342	265	207	163	129	103	67	45	31	21
.8	24568	17633	12674	9382	7072	5380	4112	3154	2427	1875	1454	1132	886	697	551	438	350	281	227	185	124	85	60	42
1.0	40627	19669	12946	9438	7197	5607	4417	3501	2785	2223	1778	1427	1148	927	751	611	499	409	337	278	193	135	97	70
1.2	19292	14865	11203	8635	6813	5462	4423	3604	2947	2416	1984	1633	1347	1112	921	765	636	531	445	373	266	192	140	104
1.4	11076	9926	8518	7165	5989	5005	4188	3510	2946	2475	2081	1751	1475	1243	1050	887	751	638	542	462	337	299	186	140
1.6	7463	7028	6413	5720	5626	4377	3790	2371	2818	2423	2083	1789	1536	1319	1133	974	838	722	622	537	403	304	231	177
1.8	5480	5272	4960	4588	4164	3744	3339	2959	2611	2296	2013	1763	1541	1346	1175	1026	896	782	683	597	458	353	273	213
2.0	4241	4126	3948	3721	3462	3186	2907	2633	2371	2126	1891	1692	1503	1334	1182	1046	926	819	724	640	502	394	311	246
2.2	3400	3331	3221	3076	2907	2720	2525	2326	2131	1942	1762	1594	1437	1293	1161	1041	932	834	746	667	533	427	342	275
2.4	2798	2753	2680	2584	2468	2338	2198	2053	1906	1761	1619	1484	1355	1234	1222	1017	921	833	753	680	554	451	367	294
2.6	2349	2318	2268	2201	2119	2025	1923	1815	1703	1591	1480	1371	1266	1165	1070	981	897	819	747	681	564	467	386	319
2.8	2003	1981	1945	1897	1837	1768	1692	1610	1525	1437	1349	1261	1175	1092	1012	936	864	976	732	673	566	475	398	334
3.0	1731	1714	1688	1652	1608	1556	1498	1435	1368	1299	1229	1158	1088	1019	952	888	825	766	710	657	561	478	405	344

TABLE 3.4: VALUE OF xk zz_2 x 10^5 for μ = 0.0 and D/a = 20 (+Tension, otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8
.2	+ 51	+ 42	+ 30	+ 18	+ 9	+ 3	1	3	4	4	3	3	2	2	1	1	1	1	1	0	0	0	0	0
.4	+ 195	+ 152	+101	+ 57	+ 26	+ 6	4	9	11	11	10	9	7	6	5	4	3	3	2	2	1	1	1	0
.6	+ 545	+ 358	+192	+ 86	+ 28	1	15	20	20	19	17	15	13	11	9	7	6	5	4	3	2	2	1	1
.8	+1693	+ 591	+176	+ 38	8	26	31	32	30	27	23	20	17	15	12	10	9	7	6	5	4	3	2	1
1.0	66	65	62	59	55	50	45	41	36	32	28	24	21	18	15	13	11	10	8	7	5	4	3	2
1.2	1828	723	302	158	102	75	60	50	42	36	32	27	24	21	18	15	13	11	10	9	6	5	4	3
1.4	687	496	323	208	140	100	80	60	50	42	35	31	26	23	20	17	15	13	12	10	8	6	5	3
1.6	349	299	239	184	140	107	84	68	55	46	39	34	29	25	22	19	17	15	13	11	9	7	5	4
1.8	214	196	172	146	121	100	82	68	57	48	41	36	31	27	24	21	18	16	14	13	10	8	6	5
2.0	146	139	128	114	100	87	75	65	56	48	42	36	32	28	25	22	19	17	18	14	11	9	7	6
2.2	108	104	98	91	83	74	66	89	52	46	41	36	32	28	25	22	20	18	16	14	12	9	8	6
2.4	84	81	78	74	69	63	58	52	47	43	38	35	31	28	25	23	20	18	17	15	12	10	8	7
2.6	67	66	64	61	58	54	50	47	43	39	46	33	30	27	25	22	20	18	17	15	13	10	9	7
2.8	55	55	53	51	49	47	44	41	38	36	33	30	28	26	24	22	20	18	17	15	13	11	9	8
3.0	47	46	45	44	42	40	39	37	34	32	30	28	26	24	22	21	19	18	16	15	13	11	9	8

TABLE 3.5: VALUE OF xk zzT x 10^6 for μ = 0.1, D/a = 20 & C_1=10(+Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	1847	912	440	234	137	85	55	36	24	16	11	18	6	4	3	2	2	1	1	1	0	0	0	0
.4	1304	1034	749	520	357	245	169	118	83	59	42	30	22	16	12	9	7	5	4	3	2	1	1	1
.6	1331	1130	893	667	502	368	270	198	145	108	80	60	45	34	26	20	16	12	10	8	5	3	2	2
.8	2030	1449	1031	759	570	432	329	251	193	148	115	89	69	54	43	34	27	21	17	14	9	6	4	3
1.0	3341	1605	1051	763	580	450	353	279	222	176	141	112	90	73	59	47	39	32	26	21	15	10	7	5
1.2	1600	1225	917	703	551	441	356	289	235	192	158	129	106	88	72	60	50	41	34	29	20	15	11	8
1.4	916	819	700	586	488	406	338	283	236	198	166	139	117	98	83	70	59	50	42	36	26	19	14	11
1.6	616	579	527	468	410	356	307	264	227	195	167	143	122	105	90	77	66	57	49	42	31	24	18	14
1.8	451	433	407	375	340	305	270	240	211	185	162	141	123	108	94	82	71	62	54	47	36	28	21	17
2.0	348	338	323	304	283	260	236	214	192	172	153	136	121	107	94	83	74	65	57	51	40	31	24	19
2.2	279	273	264	251	237	222	205	189	173	157	142	128	116	104	93	83	74	66	59	53	42	34	27	22
2.4	229	225	219	211	201	190	179	167	155	143	131	120	109	99	90	82	74	66	60	54	44	36	29	24
2.6	192	189	185	179	173	165	156	147	138	129	120	111	99	94	86	77	72	66	60	54	45	37	31	25
2.8	163	162	159	155	150	144	138	131	124	116	119	102	95	88	82	75	69	64	59	54	45	38	32	26
3.0	141	140	137	134	131	126	122	116	111	105	199	94	88	82	77	71	66	62	57	53	45	38	32	25

TABLE 3.6: VALUE OF xk zzT x 10^6 for μ = 0.1, D/a = 20 and C_1=0.0 (+Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	+ 172	+ 140	+ 99	+ 59	+ 28	+ 8	5	11	12	12	11	9	7	6	5	4	3	2	2	1	1	1	0	0
.4	+657	+ 509	+336	+487	+ 82	+ 17	18	34	38	37	33	29	24	20	16	13	10	8	7	5	3	2	1	1
.6	+1828	+1192	+630	+275	+ 83	12	54	69	70	66	58	50	40	35	29	23	19	16	13	10	7	5	3	2
.8	+5637	+1922	+548	+109	40	93	108	107	99	89	77	67	57	48	40	33	28	23	19	16	11	8	6	4
1.0	218	213	204	193	179	165	149	134	118	104	90	78	67	57	49	42	35	30	25	22	16	11	8	6
1.2	6082	2356	963	500	321	237	190	158	135	116	101	87	75	65	56	49	42	36	31	27	20	15	11	8
1.4	2299	1649	1064	679	453	322	243	192	158	132	113	97	84	73	63	55	48	41	36	31	24	18	14	11
1.6	1167	999	794	607	459	350	273	218	178	148	125	107	92	80	69	61	53	46	41	36	28	21	17	13
1.8	714	654	572	483	400	328	269	222	186	156	133	114	99	86	75	66	58	51	45	40	41	24	19	15
2.0	489	463	424	379	332	287	247	211	181	156	135	117	102	90	79	70	62	55	48	43	34	27	22	17
2.2	360	347	326	301	274	245	218	193	170	150	132	116	103	91	81	72	69	57	51	46	37	36	24	19
2.4	278	271	259	244	227	209	190	172	155	140	125	112	101	90	81	73	65	59	53	48	39	32	26	21
2.6	223	219	211	202	191	179	166	153	140	128	117	106	96	87	79	72	65	59	54	49	40	33	27	23
2.8	184	181	176	170	162	154	185	135	126	117	108	99	91	83	76	70	64	59	53	49	41	34	29	24
3.0	154	152	149	145	140	134	127	120	113	106	99	92	85	79	73	67	62	57	53	48	41	35	29	25

TABLE 3.7: VALUE OF xk $zz_1 \times 10^5$ for $\mu = 0.1$, D/a = 20 (+ Tension otherwise compression)

Q/S 666	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8
.2	23216	11463	5523	2944	1719	1066	686	453	304	207	143	99	70	50	40	26	19	14	11	8	5	3	2	1
.4	16389	12996	9412	6540	4487	3081	2128	1482	1042	738	528	382	278	205	152	114	86	66	51	39	24	15	10	7
.6	16725	14201	11223	8504	6304	4620	3388	2484	1828	1352	1007	754	569	432	331	255	198	155	122	97	62	41	28	19
.8	25509	18157	12955	9537	7157	5423	4131	3157	2421	1863	1439	1117	870	682	537	425	338	271	218	176	118	80	55	39
1.0	41980	20171	13210	9590	7285	5657	4442	3510	2784	2214	1766	1413	1133	912	736	597	486	397	325	268	184	129	91	65
1.2	20110	15395	11524	8831	6934	5537	4468	3628	2957	2417	1980	1624	1336	1100	908	752	624	519	433	363	257	184	134	98
1.4	11515	10288	8793	7364	6128	5100	4252	3551	2971	2488	2086	1750	1470	1236	1041	878	741	627	532	452	329	241	179	134
1.6	7735	7273	6619	5887	5156	4475	3863	3323	2854	2447	2097	1797	1539	1318	1130	969	832	715	615	530	395	297	225	111
1.8	5665	5445	5114	4711	4214	3834	3410	3014	2652	2326	2035	1777	1550	1351	1177	1025	893	778	678	592	452	347	267	208
2.0	4374	4253	4065	3826	3553	3264	2971	2686	2413	2159	1924	1710	1517	1343	1187	1049	926	817	722	637	497	389	306	241
2.2	3501	3428	3311	3160	2982	2786	2581	2575	2171	1975	1789	1615	1453	1305	1170	1047	936	836	746	666	531	423	338	271
2.4	2876	2829	2752	2651	2530	2394	2247	2046	1943	1792	1645	1505	1372	1248	1132	1025	927	837	755	681	553	448	364	296
2.6	2411	2378	2326	2255	2170	2072	1965	1853	1737	1620	1504	1392	1283	1180	1082	990	904	824	751	683	564	466	384	317
2.8	2053	2030	1993	1942	1880	1808	1728	1643	1554	1463	1372	1281	1193	1107	1024	946	870	820	737	676	568	475	397	332
3.0	1772	1755	1727	1690	1643	1589	1529	1463	1394	1323	1250	1177	1104	1033	964	898	834	773	716	662	564	479	405	343

TABLE 3.8: VALUE OF xk $zz_2 \times 10^5$ for $\mu = 0.1$, D/a = 20 (+ Tension otherwise compression)

666

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	+ 54	+ 44	+ 31	+ 19	+ 9	+ 2	1	3	4	4	3	3	2	2	51	1	1	1	1	0	0	0	0	0
.4	+ 207	+160	+105	+ 59	+ 26	+ 5	6	11	12	12	11	9	7	6	5	4	3	3	2	2	1	1	0	0
.6	+ 574	+375	+198	+ 89	+ 26	4	17	22	22	21	18	16	13	11	9	7	6	5	4	3	2	1	1	1
.8	+1771	+604	+172	+ 34	13	29	34	38	31	28	24	21	18	15	13	11	9	7	6	5	4	2	2	1
1.0	69	67	64	61	56	52	47	42	37	33	28	25	21	18	15	13	11	9	8	7	5	4	3	2
1.2	1910	740	302	157	101	74	60	50	42	37	32	27	24	21	18	15	13	11	10	8	6	5	3	3
1.4	722	518	534	213	142	101	76	60	50	42	35	30	26	23	20	17	15	13	11	10	7	6	4	3
1.6	367	314	250	191	144	110	86	68	56	46	39	34	29	25	22	19	17	15	13	11	9	7	5	4
1.8	224	206	180	152	126	103	85	70	58	49	42	36	31	27	24	21	18	16	14	12	10	8	6	5
2.0	154	145	133	119	104	90	77	66	57	49	42	37	32	28	25	22	19	17	15	14	11	9	7	5
2.2	113	109	103	95	86	77	69	61	53	47	41	37	32	29	25	23	20	18	16	14	12	9	8	6
2.4	87	85	81	77	71	66	60	54	49	44	39	35	32	28	25	23	21	18	17	15	12	10	8	7
2.6	70	69	66	63	60	56	52	48	44	40	37	33	30	27	25	23	20	19	17	15	13	10	9	7
2.8	58	57	55	53	51	58	45	43	40	37	34	31	29	26	24	22	20	18	17	15	13	11	9	8
3.0	49	48	47	46	44	42	40	38	35	33	31	29	27	25	23	21	20	18	17	15	13	11	9	8

TABLE 3.9: VALUE OF xk zzT x 10^6 for μ = .3, D/a = 20 and C_1 = 1(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	1854	923	448	240	141	87	56	36	24	16	11	7	5	3	2	2	1	1	1	0	0	0	0	0
.4	1344	1068	775	539	369	252	172	119	82	57	40	29	20	15	11	8	6	4	3	2	1	1	0	0
.6	1427	1204	943	708	520	378	274	199	144	105	77	57	42	32	24	18	14	10	8	6	4	2	2	1
.8	2244	1564	1095	794	539	442	333	252	191	146	111	85	66	51	39	31	24	19	15	12	8	5	3	2
1.0	3648	1719	1111	798	600	461	359	281	221	174	138	109	87	69	85	44	36	29	23	19	13	9	6	4
1.2	1786	1346	990	747	579	458	366	294	238	193	156	127	104	85	69	57	47	39	32	26	18	13	9	7
1.4	1016	901	762	631	519	428	353	292	242	201	167	139	116	97	81	68	57	48	40	34	24	18	13	9
1.6	677	634	574	506	440	378	324	276	235	200	180	145	123	105	89	76	65	55	47	40	30	22	16	12
1.8	493	473	442	405	365	325	287	252	220	192	167	145	125	109	94	81	70	61	53	46	35	26	20	15
2.0	378	367	350	328	304	277	251	226	202	179	158	140	124	109	96	84	74	65	57	50	39	30	23	18
2.2	301	295	284	270	254	237	218	200	182	165	148	133	119	107	95	85	75	67	59	53	42	33	26	21
2.4	247	242	235	226	215	203	190	177	163	150	137	125	113	102	92	83	75	67	61	54	44	35	28	23
2.6	206	203	198	192	184	176	166	156	146	135	125	116	108	97	88	81	74	67	61	55	45	37	30	25
2.8	175	173	169	165	159	153	147	138	130	122	114	106	99	91	84	76	71	65	60	55	46	38	31	26
3.0	150	149	146	143	139	134	129	123	117	111	104	98	92	85	79	74	68	63	58	54	45	38	32	27

TABLE 3.10 VALUE OF xk zzT x 10^6 for μ = .3, D/a = 20 and C_1=0.0 (+Tension otherwise compression

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	22	2.4	2.6	2.8
.2	+ 201	+ 162	+ 113	+ 66	+30	+ 6	8	14	16	15	13	11	9	7	5	4	3	2	2	1	1	1	0	0
.4	+ 760	+ 584	+ 378	+204	+82	+ 9	29	45	48	45	40	33	27	22	17	14	11	8	7	5	3	2	1	1
.6	+ 2092	+1343	+ 689	+280	+67	34	76	88	86	78	67	56	47	38	31	25	20	16	13	10	7	4	3	2
.8	+6345	+2037	+515	+ 60	88	127	135	128	115	100	86	73	61	51	42	34	28	23	19	16	10	7	5	3
1.0	241	234	224	211	196	179	161	144	126	110	95	82	70	59	50	42	35	29	25	21	14	10	7	5
1.2	6836	2515	971	488	312	232	188	158	136	117	101	88	76	65	56	48	41	35	30	25	18	13	10	7
1.4	2620	1849	1165	724	472	329	245	193	158	132	112	96	83	72	62	54	46	40	34	30	22	17	12	9
1.6	1330	1129	888	668	497	373	286	225	182	150	136	107	92	79	68	59	52	45	39	34	26	20	15	12
1.8	811	740	642	537	440	357	289	236	195	162	137	117	100	86	75	65	57	50	44	38	30	23	18	14
2.0	554	522	477	423	368	315	268	228	194	165	142	122	106	92	80	70	62	84	48	42	33	26	20	16
2.2	406	390	366	336	304	271	239	210	183	160	140	122	107	94	83	73	65	58	51	45	36	29	23	18
2.4	313	304	290	272	252	231	209	188	169	150	134	119	106	94	84	75	67	60	53	48	38	31	25	20
2.6	250	244	236	225	212	197	182	167	153	139	126	113	102	92	83	75	67	61	55	49	40	43	27	22
2.8	205	201	196	188	180	170	159	148	137	128	116	106	97	89	81	73	67	61	55	50	41	34	28	23
3.0	172	169	165	160	154	147	139	131	123	115	107	99	9[illegible]	84	77	71	65	60	55	50	42	45	29	25

TABLE 3.11: VALUE OF xk $zz_1 \times 10^5$ for μ = 0.3, D/a=20 (+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8
.2	23292	11598	5623	3019	1766	1093	699	456	301	201	135	92	63	43	30	21	15	10	7	5	2	1	1	0
.4	16891	13427	9744	6770	4633	3163	2167	1492	1034	722	507	359	256	184	133	97	71	53	39	29	17	10	6	4
.6	17930	15124	11850	8897	6534	4750	3442	2495	1814	1325	992	717	532	398	299	226	172	132	102	79	48	30	19	13
.8	28197	19653	13759	9981	7401	5549	4184	3166	2404	1830	1398	1072	825	638	496	387	304	239	190	151	97	64	43	29
1.0	45846	21607	13964	10023	7537	5798	4512	3534	2779	2191	1731	1372	1090	868	694	557	448	362	293	239	160	109	76	53
1.2	22446	16909	12441	9392	7281	5750	4594	3696	2987	2421	1966	1600	1304	1065	872	715	588	485	401	333	231	163	116	84
1.4	12768	11324	9579	7930	6526	5373	4434	3669	3043	2527	2101	1749	1458	1217	1016	851	713	599	504	425	304	220	161	119
1.6	8513	7971	7210	6363	5527	4755	4069	3472	2957	2516	2140	1819	1547	1316	1120	954	813	694	593	507	313	277	207	156
1.8	6194	5938	5554	5090	4590	4090	3613	3171	2770	2413	2096	1818	1576	1364	1181	1022	884	766	664	576	434	329	251	193
2.0	4756	4616	4399	4125	3814	3486	3156	2836	2534	2253	1996	1764	1555	1368	1203	1057	928	814	715	628	485	375	292	228
2.2	3787	3704	3571	3398	3195	2974	2744	2512	2286	2069	1864	1674	1499	1339	1194	1063	946	840	747	663	523	413	327	260
2.4	3098	3044	2957	2843	2705	2552	2388	2218	2048	1881	1720	1566	1422	1287	1162	1048	943	848	761	683	550	442	356	287
2.8	2587	2550	2491	2412	2315	2206	2086	1960	1832	1703	1575	1452	1333	1221	1115	1016	924	840	762	690	566	463	379	310
2.8	2196	2170	2128	2071	2001	1921	1832	1737	1639	1538	1438	1338	1242	1148	1059	975	895	820	751	687	572	476	395	328
3.0	1889	1870	1839	1797	1746	1685	1618	1546	1469	1390	1310	1230	1151	1074	999	927	858	793	732	675	571	482	405	341

TABLE 3.12: VALUE OF xk $zz_2 \times 10^5$ for $\mu = 0.3$, D/a = 20 (+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	+ 63	+ 61	+ 34	+ 21	+ 9	+ 2	3	4	5	5	4	3	3	2	2	1	1	1	1	0	0	0	0	0
.4	+ 239	+183	+119	+ 64	+ 26	+ 3	9	14	15	14	12	10	9	7	5	4	3	3	2	2	1	1	0	0
.6	+ 659	+422	+216	+ 88	+ 21	11	24	28	27	24	21	18	15	12	10	8	6	5	4	3	2	1	1	1
.8	+1993	+640	+162	+119	26	40	42	40	36	32	27	23	19	16	13	11	9	7	6	5	3	2	2	1
1.0	76	74	71	66	62	56	51	45	40	35	30	26	22	19	16	13	11	9	8	7	5	3	2	2
1.2	2148	790	305	153	98	73	59	50	43	37	32	28	24	20	18	15	13	11	9	8	6	4	3	2
1.4	823	581	366	228	148	103	77	61	50	41	35	30	26	23	19	17	15	13	11	9	7	5	4	3
1.6	418	355	279	210	156	117	90	71	57	47	40	34	29	25	22	19	16	14	12	11	8	6	5	3
1.8	255	233	202	169	138	112	91	74	61	51	43	37	31	27	24	21	18	16	14	12	9	7	6	4
2.0	174	164	150	133	116	99	84	72	61	52	44	38	33	29	25	22	19	17	15	13	10	8	6	5
2.2	128	123	115	106	95	85	75	66	58	50	40	38	34	30	26	23	20	18	16	14	11	9	7	6
2.4	98	96	91	86	79	73	66	59	53	47	42	37	33	30	26	24	21	19	17	15	12	10	8	6
2.6	78	77	74	71	66	62	57	53	48	44	39	36	32	29	26	24	21	19	17	16	13	10	8	7
2.8	64	63	62	59	56	53	50	47	43	40	37	33	31	38	25	23	21	19	17	16	13	11	9	7
3.0	54	53	52	50	48	46	44	41	39	36	34	31	29	26	24	22	20	19	17	16	13	11	9	8

TABLE 3.13 : VALUE OF xk zzT x 10^6 for μ = .5, D/a = 20 and C_1=1(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	1864	942	464	251	147	91	57	37	24	15	10	6	4	2	2	1	1	0	0	0	0	0	0	0
.4	1416	1130	823	572	390	263	178	120	81	55	37	25	17	12	8	5	4	2	1	1	0	0	0	0
.6	1599	1336	1033	764	553	396	282	200	142	101	27	52	37	27	19	14	10	7	5	4	2	1	0	0
.8	2629	1778	1210	858	624	459	341	253	189	140	105	79	59	45	34	25	19	15	11	8	5	3	2	1
1.0	4202	1925	1219	860	636	482	369	265	220	171	133	103	81	63	49	38	30	24	19	15	9	6	4	3
1.2	2121	1562	1121	828	629	488	384	304	241	193	155	124	99	80	64	52	42	34	27	22	15	10	7	5
1.4	1196	1049	875	712	576	467	379	309	252	207	169	139	114	94	97	64	53	44	36	30	21	14	10	7
1.6	789	734	658	575	493	419	353	298	250	210	176	148	124	104	88	74	62	52	44	37	27	19	14	10
1.8	569	543	505	459	411	362	317	275	237	204	176	151	129	110	95	88	69	59	51	43	32	24	18	13
2.0	342	334	321	304	285	264	242	220	198	178	159	142	126	111	99	87	77	68	59	52	42	31	24	19
2.4	278	273	265	254	240	226	210	194	178	163	148	133	120	108	97	87	77	69	61	55	43	34	27	22
2.6	231	227	222	214	205	195	183	191	159	147	136	124	113	103	94	85	76	69	62	56	45	36	29	24
2.8	195	192	189	183	177	169	161	152	143	133	124	115	106	97	89	82	75	68	62	56	46	38	31	25
3.0	167	165	162	158	154	188	142	135	128	120	113	106	98	91	84	78	72	66	61	56	46	39	32	27

TABLE 3.14: VALUE OF xk zzT x 10^6 for μ = 0.5, D/a = 20 and C_1 = 0.0 (+Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8
.2	+ 252	+ 202	+ 138	+ 79	+ 23	+ 3	14	23	21	20	27	13	111	8	8	5	3	3	2	1	1	0	0	0
.4	+ 945	+ 717	+ 454	+234	+ 83	5	49	65	66	59	51	41	33	26	20	15	12	9	7	5	3	1	1	0
.6	+2581	+1615	+ 788	+289	+ 37	75	116	122	114	99	83	68	55	44	35	27	21	16	13	10	6	3	2	1
.8	+7619	+2242	+ 455	30	161	188	182	166	143	121	102	84	69	56	45	36	29	23	18	15	9	6	3	2
1.0	280	273	261	244	226	205	183	162	141	122	104	88	74	62	51	42	35	28	23	19	12	8	5	3
1.2	8193	2800	986	467	295	224	185	158	137	119	103	89	76	65	55	46	39	33	27	23	16	11	7	5
1.4	3196	2208	1348	806	505	342	250	195	158	132	111	95	81	70	60	51	44	37	32	27	19	14	10	7
1.6	1622	1364	1055	780	565	416	310	229	190	154	128	107	91	78	67	57	49	42	36	31	23	17	12	9
1.8	987	895	769	635	512	408	325	261	211	173	144	121	102	87	75	64	55	48	41	36	27	20	15	11
2.0	670	630	571	502	432	366	307	287	215	181	153	130	111	95	82	71	61	53	46	40	31	23	18	14
2.2	489	468	438	399	358	316	276	239	207	179	154	133	115	100	87	76	66	58	51	45	35	27	21	16
2.4	375	363	346	323	297	270	243	216	192	270	149	132	116	102	90	79	70	62	54	48	38	30	23	19
2.6	298	291	280	266	249	231	212	193	175	157	141	126	113	101	90	80	72	64	57	51	40	32	26	21
2.8	243	239	231	222	211	198	185	171	157	144	131	119	108	98	88	80	72	65	58	52	42	34	28	23
3.0	203	200	195	188	180	171	162	152	141	131	121	111	102	93	85	78	71	64	58	53	44	36	29	24

TABLE 3.15: VALUE OF $xk\ zz_1 \times 10^5$ for $\mu = 0.5$, $D/a = 20$ (+ Tension otherwise compression)

Z/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8
.2	23430	11839	5829	3156	1853	1141	721	461	296	190	122	78	50	31	19	11	6	3	1	0	+1	+ 2	+2	+ [illegible]
.4	17795	14202	10340	7185	4895	3311	2235	1510	1021	692	469	319	216	147	99	67	44	29	19	12	3	0	+2	+ [illegible]
.6	20094	16785	12978	9604	6948	4970	3539	2516	1790	1275	910	651	466	335	241	173	125	90	65	46	23	11	5	1
.8	33037	22346	15205	10778	7838	5774	4280	3183	2372	1770	1324	992	744	560	422	319	241	183	139	106	62	36	21	12
1.0	52803	24190	15322	10802	7990	6052	4639	3578	2770	2148	1669	1298	1012	789	617	483	380	299	136	186	117	75	48	31
1.2	26651	19634	14091	10400	7905	6134	4821	3820	3040	2427	1942	1555	1248	1002	806	650	524	424	344	279	185	124	84	58
1.4	25025	13188	10993	8951	7241	5863	4763	3882	3172	2597	2129	1747	1436	1181	972	801	661	547	453	375	260	181	128	91
1.6	9914	9229	8274	7221	6195	5260	4441	3739	3142	2639	2216	1860	1561	1311	1102	926	779	656	553	467	334	240	174	128
1.8	7146	6826	6347	5772	5159	4551	3977	3453	2983	2569	2207	1893	1622	1388	1188	1016	870	744	638	546	403	298	222	166
2.0	5442	5269	5001	4663	4283	3885	3489	3108	2751	2423	2126	1860	1623	1415	1231	1071	931	809	703	611	462	350	267	204
2.2	4303	4200	4037	3826	3580	3313	3036	2760	2492	2238	20[illegible]1	1781	1582	1401	1238	1093	963	849	747	658	509	395	307	240
2.4	3498	3432	3327	3187	3022	2837	2641	2434	2238	2042	1854	1677	1511	1357	1217	1088	972	867	772	688	545	431	342	271
2.6	2905	2860	2788	2693	2577	2446	2303	2154	2026	1851	1703	1560	1414	1295	1175	1064	961	867	781	703	568	458	369	298
2.8	2453	2422	2371	2303	2220	2124	2019	1907	1791	1673	1556	1441	1330	1223	1122	1026	937	853	776	705	581	477	391	320
3.0	2100	2078	2041	1991	1930	1858	1779	1694	1604	1512	1419	1326	1235	1147	1061	980	903	830	762	698	584	487	405	337

TABLE 3.16: VALUE OF $xk\ zz_2 \times 10^5$ for $\mu = 0.5$, $D/a = 20$ (+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	
.2	+ 79	+ 63	+ 43	+ 25	+ 10	+ 1	4	6	7	6	5	4	3	3	2	1	1	1	1	0	0	0	0	0	0
.4	+ 270	+225	+143	+ 73	+ 26	2	15	20	21	19	16	13	10	8	6	5	4	3	2	2	1	0	0	0	
.6	+ 811	+307	+247	+ 91	+ 12	24	36	38	36	31	26	21	17	14	11	8	7	5	4	3	2	1	1	0	
.8	+2393	+704	+143	9	50	59	57	52	45	38	32	26	22	18	14	11	9	7	6	5	3	2	1	1	
1.0	88	86	82	77	71	64	58	51	44	38	33	28	23	19	16	13	11	9	8	6	4	3	2	1	
1.2	2574	880	310	147	73	70	58	50	43	37	32	28	24	20	17	15	12	10	9	7	5	3	2	2	
1.4	1004	694	423	253	159	108	79	61	50	41	35	30	26	22	19	16	14	12	10	8	6	4	3	2	
1.6	510	429	331	244	177	130	97	75	60	39	40	44	29	24	21	18	15	13	11	10	7	5	4	3	
1.8	310	281	242	200	161	128	102	82	66	54	45	38	32	27	23	20	17	15	13	11	8	6	5	4	
2.0	210	198	179	158	136	115	96	81	68	57	48	41	35	30	36	22	19	17	15	13	10	7	6	4	
2.2	154	147	137	125	112	99	87	75	65	56	48	42	36	31	27	24	21	18	16	14	11	8	7	5	
2.4	118	114	109	101	93	85	76	68	60	58	47	41	36	32	28	25	22	19	17	15	12	9	7	6	
2.6	94	91	88	83	78	72	67	61	55	49	44	40	35	32	28	25	22	20	18	16	13	10	8	7	
2.8	76	75	73	70	66	62	58	54	49	45	41	37	34	31	28	25	23	20	18	16	13	11	9	7	
3.0	64	63	61	59	57	54	51	48	44	41	38	35	32	29	27	24	22	20	18	17	14	11	9	8	

TABLE 3.17: VALUE OF PkzzT x 10^3 for μ = 0.1, D/a=20 and C_1=1.0(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.[illegible]	2.[illegible]	2.6	2.8	2.9
.2	739	365	176	94	55	40	22	14	10	7	5	3	2	2	1	1	1	0	0	0	0	0	0	0	0	0
.4	522	414	300	208	143	98	68	47	33	24	17	12	9	7	5	4	3	2	2	1	1	0	00	0	0	0
.6	532	452	357	271	201	147	108	79	58	43	32	24	18	14	11	8	6	5	4	3	2	1	1	1	1	0
.8	812	578	412	304	228	173	131	100	77	59	46	36	28	22	17	14	11	9	7	6	4	3	2	2	1	1
1.0	1336	642	420	305	232	180	141	112	87	70	56	45	36	29	23	19	15	13	10	9	6	4	3	3	2	2
1.2	640	490	369	281	221	176	142	115	94	77	63	52	43	35	29	24	20	17	14	12	8	6	5	4	3	3
1.4	367	237	280	234	195	162	135	113	95	79	66	56	47	39	33	28	24	20	17	14	10	8	7	6	4	4
1.6	246	231	211	187	166	142	123	106	91	78	67	57	49	42	36	31	26	23	20	17	13	9	8	7	5	5
1.8	180	173	163	150	136	122	109	96	84	74	65	57	49	43	37	33	28	25	22	19	14	11	10	9	7	6
2.0	139	135	129	122	113	104	95	85	77	69	61	54	48	43	38	33	29	26	23	20	16	12	11	10	8	7
2.2	111	109	105	101	96	87	82	76	69	63	57	51	46	42	37	33	30	27	24	21	17	13	12	11	9	8
2.4	92	90	88	84	81	76	72	67	62	57	52	48	44	40	36	33	30	27	24	22	18	14	13	12	9	9
2.6	77	76	74	72	69	66	63	59	55	47	44	41	38	35	33	30	28	26	23	22	18	15	14	13	11	10
2.8	65	65	63	62	60	58	55	52	49	47	44	41	38	35n	33	30	28	26	23	22	18	15	14	13	11	10
3.0	56	56	55	54	52	51	49	47	44	42	38	37	35	33	31	29	27	25	23	21	18	15	14	13	11	10

TABLE 3.18: VALUE OF xk zzT x 10^6 for $\mu = 0.1$, D/a=40 and $C_1 = 1.0$
(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.2	475	225	108	58	34	21	14	9	6	4
.4	330	260	188	130	89	61	42	29	21	15
.6	336	284	224	169	125	92	67	49	36	27
.8	517	362	257	189	142	108	82	63	48	37
1.0	794	399	262	190	145	112	88	70	53	44
1.2	406	307	229	176	138	110	89	72	59	48
1.4	231	206	175	147	122	101	85	71	59	50
1.6	155	145	132	117	103	89	77	66	57	49
1.8	113	109	102	94	85	76	68	60	53	46
2.0	87	85	81	76	71	65	59	53	48	43
2.2	70	68	66	63	59	56	51	47	43	39
2.4	57	56	55	53	50	48	45	42	39	36
2.6	48	47	46	45	43	41	39	37	35	32
2.8	41	40	40	39	37	36	34	33	31	29
3.0	35	35	34	34	33	32	30	29	28	26

TABLE 3.19: VALUE OF xk zzT x 10^6 for μ = 0.1, D/a=40, C_1=0.0
(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.2	+ 43	+ 35	+ 25	+ 15	+ 7	+ 2	1	3	3	3
.4	+ 166	+ 128	+ 84	+ 47	+ 20	+ 4	5	9	10	9
.6	+ 462	+ 299	+ 157	+ 68	+ 20	3	14	17	18	16
.8	+1430	+ 473	+ 134	+ 26	11	24	27	27	25	22
1.0	55	53	51	48	45	41	37	33	30	26
1.2	1542	581	238	124	80	59	47	40	34	29
1.4	580	413	266	169	113	80	61	48	39	33
1.6	293	250	199	152	115	87	68	54	44	37
1.8	179	164	143	121	100	82	67	56	46	39
2.0	122	116	106	95	83	72	62	53	45	39
2.2	90	87	82	75	68	61	55	48	43	37
2.4	70	68	65	61	57	52	48	43	39	35
2.6	56	55	53	51	48	45	41	38	35	32
2.8	46	45	44	42	41	38	36	34	32	29
3.0	39	38	37	36	35	33	32	30	28	26

TABLE 3.20 : VALUE OF xk zzT x 10^6 for μ=0.1, D/a=80 and C_1=1.0
(+ Tension otherwise compression)

Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.2	120	56	27	14	8	5	3	2	2	1
.4	83	65	47	32	22	15	11	7	5	4
.6	84	71	56	43	31	23	17	12	9	7
.8	130	90	64	47	36	27	21	16	12	10
1.0	196	99	65	48	36	28	22	17	14	11
1.2	102	77	57	44	34	28	22	18	15	12
1.4	58	51	48	37	31	25	21	18	15	12
1.6	39	36	33	29	26	22	19	17	14	12
1.8	28	27	26	24	21	19	17	15	13	12
2.0	22	21	20	19	18	16	15	13	12	11
2.2	17	17	17	16	15	14	13	12	11	10
2.4	14	14	14	13	13	12	11	10	10	9
2.6	12	12	12	11	11	10	10	9	9	8
2.8	10	10	10	10	9	9	9	8	8	7
3.0	9	9	9	8	8	8	8	7	7	7

TABLE 3.21 : VALUE OF xk zzT x 10^6 for μ = 0.1, D/a=80 and C_1=0.0
(+Tension otherwise compression)

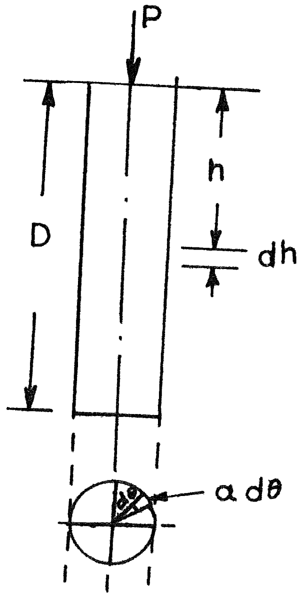
Q/S	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.2	+ 11	+ 9	+ 6	+ 4	+ 2	0	0	1	1	1
.4	+ 41	+ 32	+ 21	+ 12	+ 5	+1	1	2	2	2
.6	+116	+ 75	+ 39	+ 17	+ 5	1	3	4	4	4
.8	+359	+ 118	+ 33	+ 8	3	6	7	7	6	6
1.0	14	13	13	12	11	10	9	8	7	7
1.2	387	145	59	31	20	15	12	10	8	7
1.4	145	103	66	42	28	20	15	12	10	8
1.6	73	63	50	38	29	22	17	14	11	9
1.8	45	41	36	30	25	21	17	14	12	10
2.0	31	29	27	24	21	18	15	13	11	10
2.2	23	22	20	19	17	15	14	12	11	9
2.4	17	17	16	15	14	13	12	11	10	9
2.6	14	14	13	13	12	11	10	10	9	8
2.8	12	11	11	11	10	10	9	9	8	7
3.0	10	10	9	9	9	8	8	8	7	7

TABLE 3.22: VALUE OF RADIAL STRESS COEFFICIENTS xk zzT x 10^6, xk zz_1 x 10^5, xk zz_2 x 10^5 for μ = .1 and D/a=20
(+ Tension otherwise compression)

S	a	xk zzT x 10^6 for μ=.1 and C_1=1.0	xk zzT x 10^6 for μ=.1 and C_1=0.0	xk zz_1 x 10^5 FOR μ=.1	xk zz_2 x 10^5 for μ=.1
0.1	.2	+ 594	478	+7470	150
"	.4	+ 400	406	+5022	128
"	.8	+ 370	456	+4654	143
"	.6	+ 338	500	+4242	157
"	1.0	722	61	9076	191
"	1.2	+ 299	+325	+3757	+102
"	1.4	+ 193	+337	+2425	+106
"	1.6	+ 132	+174	+1664	+ 55
"	1.8	+ 97	+ 99	+1221	+ 31
"	2.0	+ 75	+ 62	+ 937	+ 20
"	2.2	+ 59	+ 42	+ 744	+ 13
"	2.4	+ 48	+ 30	+ 605	+ 9
"	2.6	+ 40	+ 22	+ 502	+ 7
"	2.8	+ 34	+ 17	+ 423	+ 5
"	3.0	+ 29	+ 13	+ 361	+

TABLE 3.23: VALUE OF xk zzT TO CHECK THE CONVERGENCE OF THE METHOD FOR μ = .3, C_1= 1.0 and D/a=20 (+Tension otherwise compression)

C_1	S	Q	N_1	xk zzT
1.0	1.0	0.2	20	0.00001601
1.0	1.0	0.2	40	0.00001609
1.0	1.0	0.2	80	0.00001614
1.0	1.0	0.4	20	0.00005744
1.0	1.0	0.4	40	0.00005736
1.0	1.0	0.4	80	0.00005733



FIG·3·1 SHOWING LENGTH DIAMETER AND FORCE P ACTING ON THE PILE

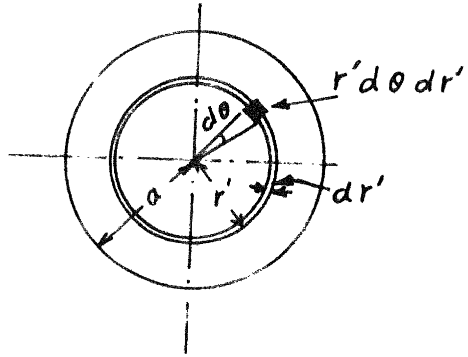
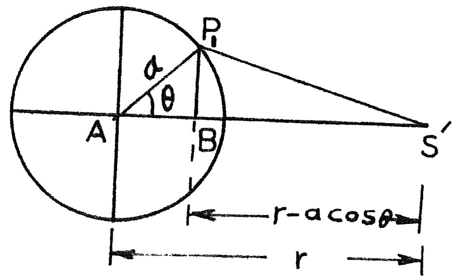


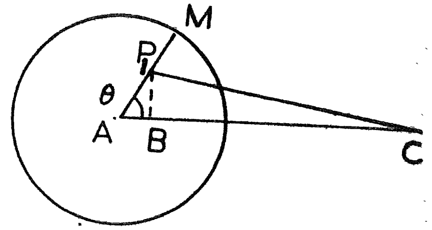
FIG. 3.2 SHOWING BASE DETAIL OF THE PILE



$AS' = r$ $\quad$ $BS' = r - a\cos\theta$

$AB = a\cos\theta$ $\quad$ $P_1B = a\sin\theta$

FIG. 3·3

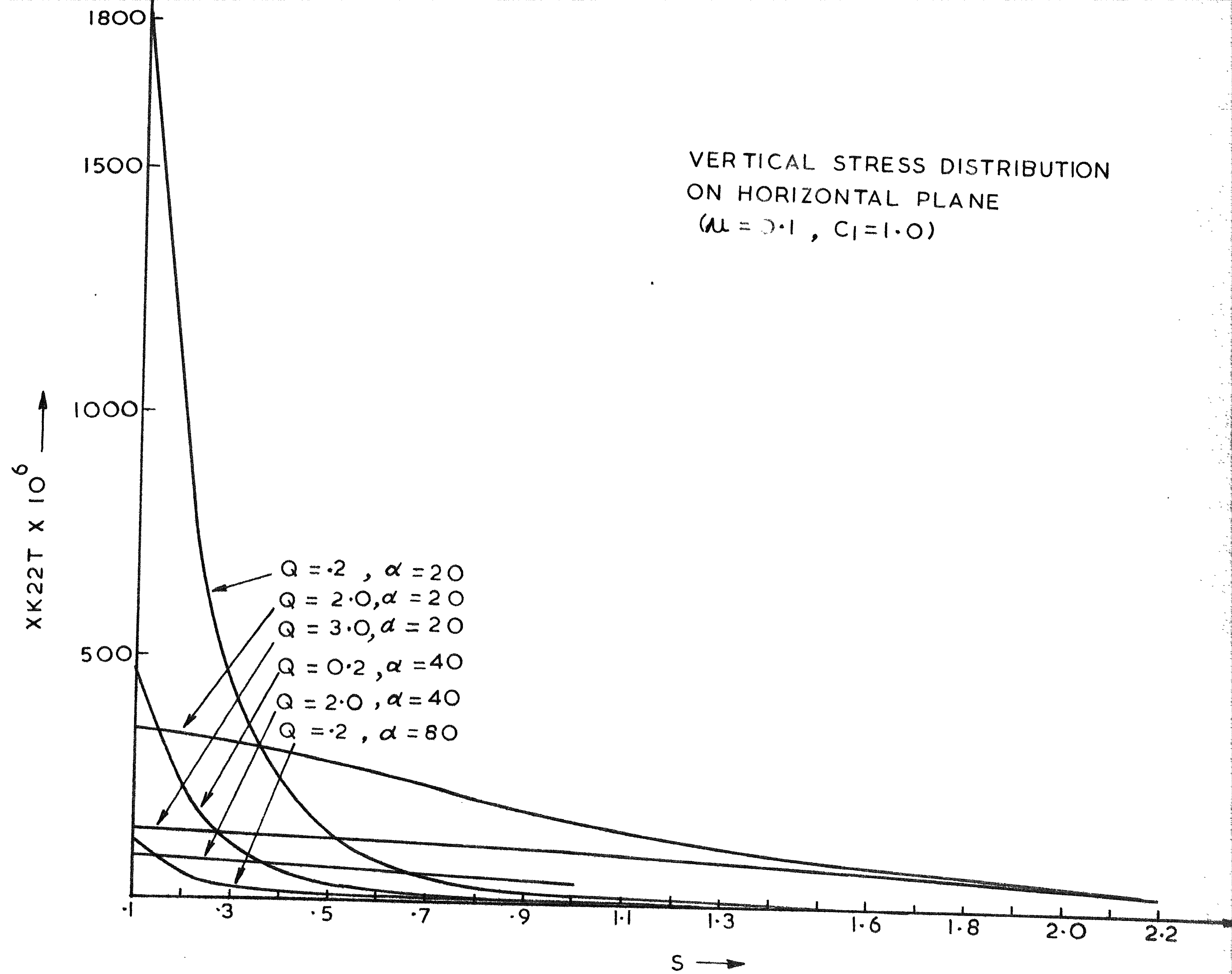


$AM = a$

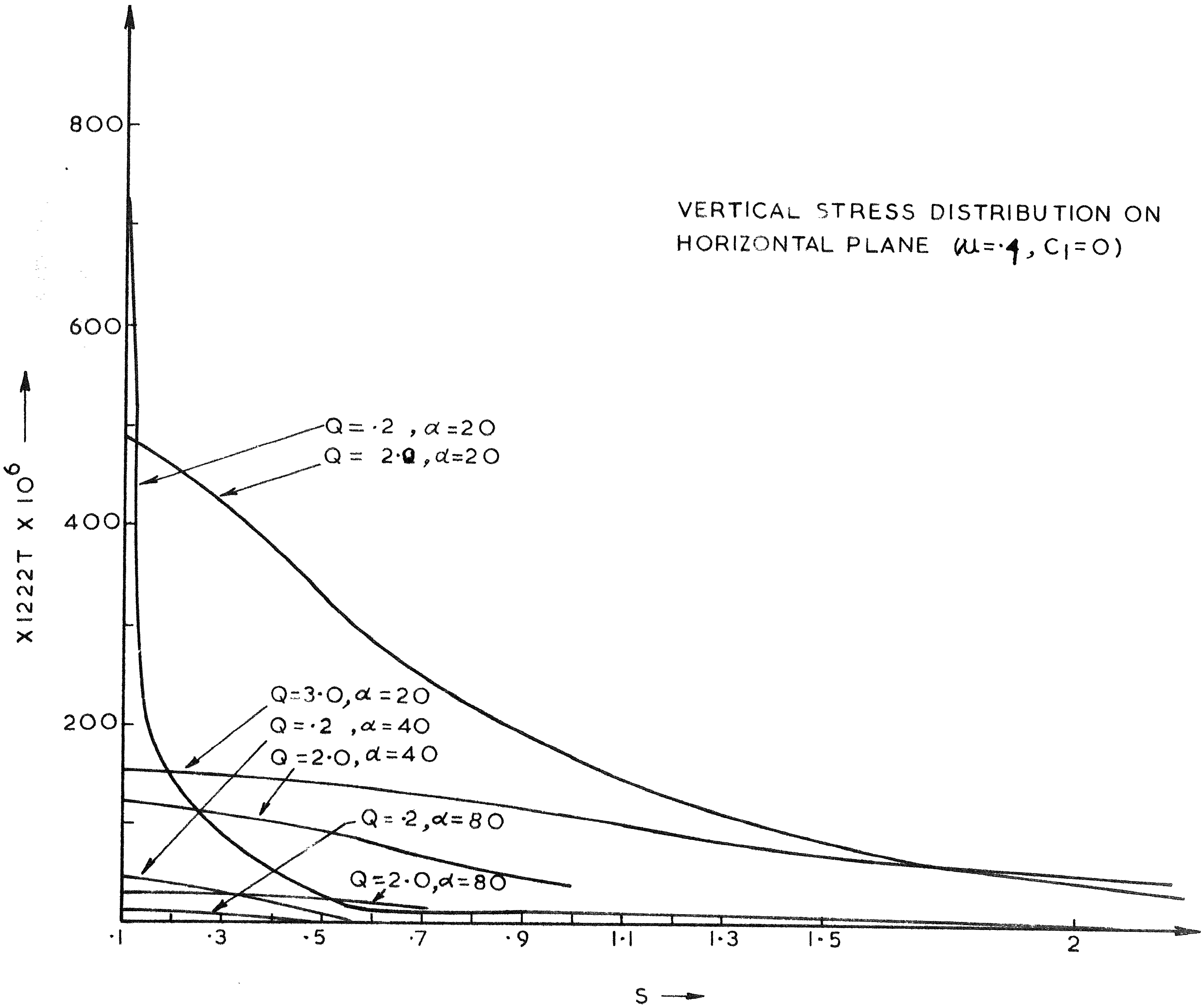
$AP_1 = r'$

$\angle P_1AB = \theta$

FIG. 3·4



VERTICAL STRESS DISTRIBUTION
ON HORIZONTAL PLANE
(μ = 0·1 , C1=1·0)
Q = ·2 , α = 20
Q = 2·0, α = 20
Q = 3·0, α = 20
Q = 0·2 , α = 40
Q = 2·0 , α = 40
Q = ·2 , α = 80
1800
1500
1000
500
XK22T X 10^6
·1
·3
·5
·7
·9
1·1
1·3
1·6
1·8
2·0
2·2
S



VERTICAL STRESS DISTRIBUTION ON
HORIZONTAL PLANE (μ=·4, C1=O)
800
600
400
200
X1222T X 10^6
Q=·2 , α=20
Q= 2·0, α=20
Q=3·O, α=20
Q=·2 , α=40
Q=2·O, α=40
Q=·2, α=80
Q=2·O, α=80
·1
·3
·5
·7
·9
1·1
1·3
1·5
2
S

CHAPTER IV

ANALYSIS OF STRESSES IN SOILS DUE TO VERTICAL LOAD ON GROUP OF PILES

4.1 INTRODUCTION:

Stresses have been found out in soils due to axially loaded single pile in Chapter III. In this chapter a general programme is developed for obtaining stresses due to orbitrary configurations of the piles. In the field load is transferred by a group of piles usually and not by a single pile. In this chapter stress coefficients are presented in tabular form for three different configurations of pile group just to illustrate the method.

4.2 FORMULATION OF PROBLEM:

Different configuration of the pile have been shown in figures 4.1, 4.2 and 4.3. In each configuration the co-ordinates of each pile and also the co-ordinates of point at which stress has to be calculated is known. In general if the co-ordinates of the point is (xk, yk, z) and co-ordinates of any pile is (x_i, y_i) then the radial distance from the centre of the pile to point is given by

$$R = ((xk - xi)^2 + (yk - yi)^2)^{\frac{1}{2}} \qquad (4.1)$$

Stress at any point will be the algebraic addition of the stress produced at that point by different piles in a group. The general programme developed for N piles gives the final stress at the point under consideration. In this programme, single pile programme is taken as subroutine to calculate the stress for any group of piles.

4.3 RESULTS:

Values for the vertical stress coefficient for above three configurations has been tabulated in Table 4.1, 4.2 and 4.3.

4.4 CONCLUSION:

Results obtained by the general programme is in total agreement with results obtained by single pile analysis.

4.5 PROGRAMME: FOR N PILES:

TABLE 4.1: VALUE OF xk zzT x 10^6 CORRESPONDING TO FIGURES 4.1, 4.2, 4.3 FOR μ = 0.1, D/a = 20 (+ Tension otherwise compression)

C_1	Co-ordinates of the points in soil (x,y,z)	Number of pile in group	xk zzT x 10^6
1.0	(4, 4, 4)	4	1292
1.0	(8, 4, 4)	4	1474
1.0	(16,4, 4)	4	562
1.0	(8, 8, 4)	4	1947
1.0	(42, 42, 60)	4	129
0.0	(4, 4, 4)	4	306
0.0	(8, 4, 4)	4	295
0.0	(16, 4, 4)	4	102
0.0	(8, 8, 4)	4	285
0.0	(42, 42, 60)	4	117
0.5	(4, 4, 4)	4	493
0.5	(8, 4, 4)	4	589
1.0	(8, 8, 4)	3	659
1.0	(16, 8, 4)	3	139
1.0	(40, 24, 4)	3	1
0.0	(8, 8, 4)	3	150
0.0	(16, 8, 4)	3	20
0.0	(40, 24, 4)	3	3
0.5	(8, 8, 4)	3	254
1.0	(8, 8, 4)	5	2778
0.0	(8, 8, 4)	5	420
0.5	(8, 8, 4)	5	.1179

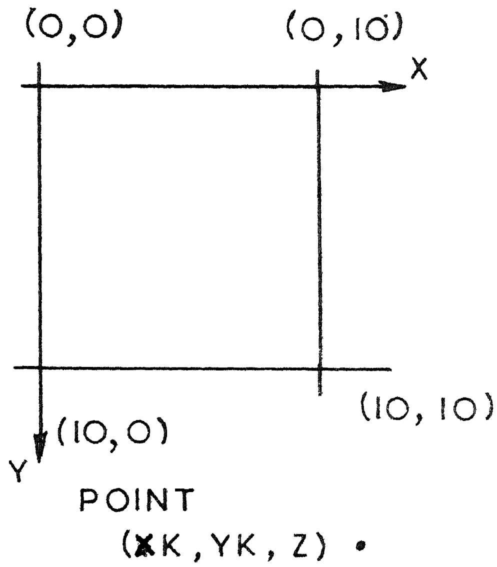


FIG. 4·1

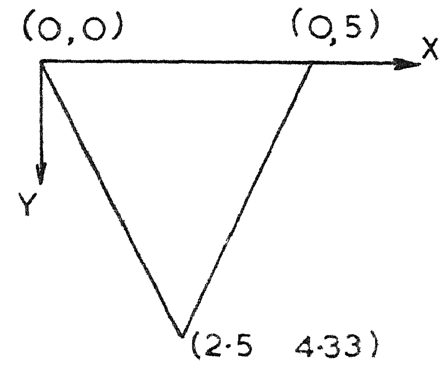


FIG. 4·2

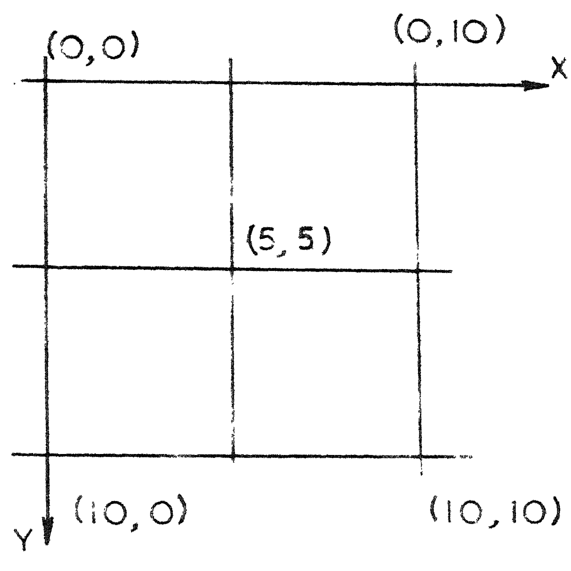


FIG. 4·3

CHAPTER V

DISCUSSIONS AND RECOMMENDATIONS

The values of stress coefficients are increasing with increase in μ of the soils. Near the pile stresses are predominant. Stresses are decreasing with increasing distance from the pile. Mostly compressive stresses are produced in case of friction pile. In case of bearing pile tension is produced in the soil above the base. Stresses are decreasing with increasing D/a. When S and Q are more than 2.5, the stresses are insignificant and produce no major changes in soils.

In Chapter III to calculate the stresses at a point in soils due to pile - type of loading Mindlin solution for a point load is used rather than Boussinesq solutions, as it is more appropriate because pile transmits its load within the soil media and not at the surface of the soil. Pile dimensions have been taken into account in formulating the problem for the stress. Thus the solution presented is more precise than Geddes. In Chapter IV the principles of Chapter III are applied to compute the stresses due to a group of pile

The only major assumption in the investigation is the value of C_1 which determines the percentage of load getting transferred to soil through the shaft and the base of the pile. Unfortunately no rigorous method exists till now to find out the value of C_1. So value of C_1 has to be determined by some methods.

Thus it can be said that the proposed investigation is a more accurate method to find out the stresses in soils due to vertical load on pile and pile-group.

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$JOB  CEG106,    TIME008,   PAGES030,   NAMEV.SINGH
$IBJOB           MAP
$IBFTC MAIN
      DIMENSION  THETA(21),PSI(21)
      COMMON/W/G,S,C1,THETA,PSI,XKZZ,XKZZ1,XKZZ2
      COMMONAA,DD,R,Z
      THETA(1)=0.0
      PSI(1)=0.0
      DO5I=2,21
      PSI(I)=PSI(I-1)+1.0/20.0
 5    THETA(I)=THETA(I-1)+0.314159
      DIMENSION A(15),D(15),X(15),Y(15)
 100  FORMAT(I2)
      READ100,NPILES
  200  FORMAT(10F5.2)
      READ200,(A(I),I=1,NPILES)
      READ200,(D(I),I=1,NPILES)
      READ200,(X(I),Y(I),I=1,NPILES)

 300  FORMAT(1H1)
      PRINT300
 400  FORMAT(10X*NPILES = *I3)
      PRINT400,NPILES
 500  FORMAT(/10X*RADII  *15F7.2)
      PRINT500,(A(I),I=1,NPILES)
 600  FORMAT(/9X*LENGTH *15F7.2)
      PRINT600,(D(I),I=1,NPILES)
 700  FORMAT(/11X*X-CO *15F7.2)
      PRINT700,(X(I),I=1,NPILES)
 800  FORMAT(/11X*Y-CO *15F7.2)
      PRINT800,(Y(I),I=1,NPILES)
      PRINT300
      DO25L=1,10
      Z=4*L
      DO35K=1,5
      YK=8*K
      DO45J=1,5
```